The Use of Spreadsheets to Calculate Water Vapour Diffusion According to the Glaser Method

Rode, Carsten

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The Use of Spreadsheets to Calculate Water Vapour Diffusion According to the Glaser Method

11102 Heat and Moisture Conditions of Buildings
Carsten Rode
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Introduction

Calculation of moisture transport by diffusion through a multi-layered construction under steady-state conditions - the so-called Glaser method - consists of two main tasks: (1) setting up of a sheet for the calculations, and (2) graphical drawing of the water vapour distribution through the construction.

Each element of the calculation is quite simple, and similarly for the drawing of the result. However, it requires a certain amount of manual work to carry out the calculation of several situation, for instance to calculate the conditions in one construction when it is exposed to different boundary conditions, or to test different variations of a construction type. Therefore it is quite natural to prepare the calculations to be carried out in a spread sheet program where changes can easily be entered, and the result seen momentarily. In addition, a spreadsheet program normally has the possibility to show the result in various different forms.

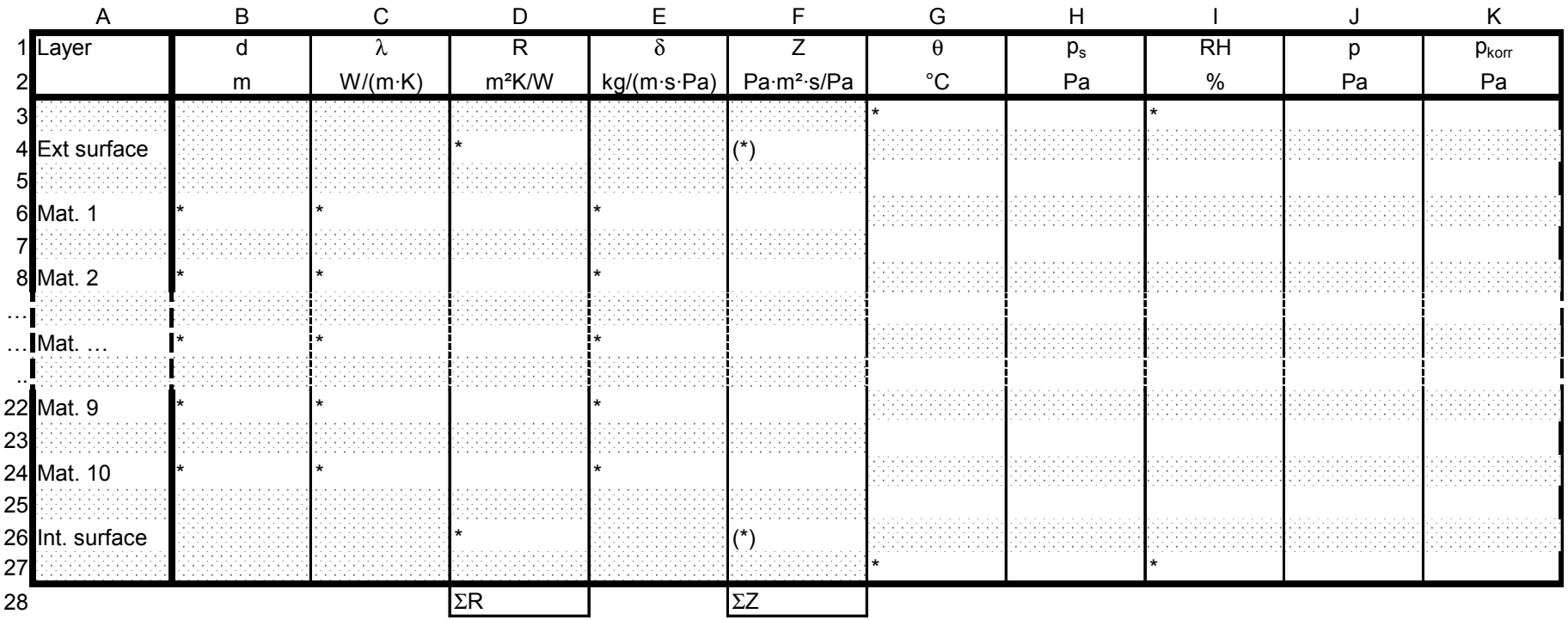
The description in this lecture note is attempted written in such a general form that it should not be necessary to have special skills in the use of spreadsheet programs to use the method. More trained users of spreadsheets can read more easily over some sections. The use of formulas in spreadsheets is written in a form that can be used in the Excel. Other spreadsheet programs may require a slightly different way to write equations. In addition, by the local settings in Windows or in the spreadsheet program, there may be used different symbols for the decimal separator, list separator, etc.

The calculation sheet that is going to be developed can be seen in its normal paper-based form in Figure 1. The cells that are filled in with an asterisk (*) should contain the initial parameters that describe a problem, i.e. the boundary conditions (temperature and relative humidity) and data about the materials in a construction. The water vapour diffusion resistances at the indoor and outdoor boundaries are normally quite small in comparison with the resistance of the real materials, so here one can usually employ the value 0 s/m. This is different from the calculation of heat flow and temperatures, where it is normally quite important to remember the thermal surface resistances. The empty cells in the sheet are the ones to be calculated.

The description in this lecture note will refer a sheet like the one in Figure 1, although it will later be necessary to introduce some additional columns to the right.

Plan for the spreadsheet

A spreadsheet consists of cells that have been placed in rows and columns in a matrix structure. The columns have an alphabetic index, while the rows are numbered with Arabic digits. Thus, a cell's position can be indicated exactly as for instance: F7. In the cells can be written characters such as letters or digits, or one can write formulas that use normal mathematical operators and functions and uses references to input from oter cells in the spreadsheet.



	A	B	C	D	E	F	G	H	I	J	K
1	Layer	d	λ	R	δ	Z	θ	p_s	RH	p	p_{korr}
2		m	W/(m·K)	m²K/W	kg/(m·s·Pa)	Pa·m²·s/Pa	°C	Pa	%	Pa	Pa
3							*		*		
4	Ext surface			*		(*)					
5											
6	Mat. 1	*	*		*						
7											
8	Mat. 2	*	*		*						
…											
…	Mat. …	*	*		*						
..											
22	Mat. 9	*	*		*						
23											
24	Mat. 10	*	*		*						
25											
26	Int. surface			*		(*)					
27							*		*		
28				ΣR		ΣZ					

Figure 1. Example of a sheet for calculation of water vapour diffusion under steady state conditions.

While it does not cause so much trouble to enter the formulas of a spreadsheet correctly from the start, it may be more difficult to check that all the formulas are correct if one is later going to insert a row for a new layer in an already existing spreadsheet. Thus, it will be an advantage to prepare the spreadsheet with rows for more layers than one really have to use for the beginning. The cells for the excess layers can filled with dummy parameters, i.e. thickness, thermal resistances and diffusion resistances with the value 0.

In many places in the spreadsheet where the number 0.0 should have been entered, one should enter a number that is very small in comparison with other numbers in the same column. Thereby, without introducing significant error, one can avoid the risk of division by 0 some places in later calculations.

First are introduced columns for the general information about a construction: First a column 'A' with the name of the materials in the layers. Then a column 'B' with the thickness, d, of the layers - it should always be written manually. Then follows a column 'C' with the thermal conductivity λ of the materials. Skip one column, and in column 'E', the water vapour permeability, δ, of the layers should be entered. In the columns for temperature, θ, and relative humidity, RH, the values for the exterior and interior boundary conditions (for the outside and inside air) should be entered in the top and bottom cells. The temperatures and RH-values in between are to be calculated.

Some of the values concern the conditions in the layers themselves, while others concern the conditions in their interfaces. In order to make this clear, a set-up is chosen where there are two rows in the spreadsheet for every material plus one (surface resistances are then included as interfaces). In addition there are rows for the titles and units. It is suggested that the layers are entered with the outside layers first (and assumed in the following).

The first column where the numbers have to be calculated using equations is the column for the thermal resistance, R, of the layers. As it should be well known, R for a layer is calculated as the thickness of the layer divided by its thermal conductivity λ. When the thermal resistance in for instance cell D6 is going to be calculated, one should enter: `=B6/C6`, and the value will now be calculated by the spreadsheet. A quite similar equation should be entered in the cells D8, D10, ... If one copies cell D6 to each of the subsequent, equal numbered cells in the same column, the spreadsheet will use the same relative placement of the cells it calls for the calculation, as in the cell it copies from. Therefore, the result of the copying becomes `=B8/C8` in cell D8, `=B10/C10` in D10, etc.

In a similar way, the water vapour diffusion resistance, Z, can be calculated in column 'F' as the thickness from column B divided by the water vapour permeability from column 'E'.

Some thermal resistances and water vapour diffusion resistances do not have to be calculated because the material thickness is very small or undefined (e.g. vapour retarder and surface resistances), or because the thermal resistance is not proportional with the layer thickness (e.g. by air gaps). In these cases, one should simply enter the correct values in stead of the calculation formula.

Now we have set up the data and formulas that have relevance only for one formula at a time. Hopefully it has been painless up till this point. Now the conditions in between the layers must be calculated, and here one has to look to the structure as a whole. Two helping quantities have to be calculated based on the now available data: The total thermal resistance of the construction, and the total water vapour diffusion resistance.

If the spreadsheet has been set up so it can hold information and calculations for 10 layers, then the first unused row will be row 28. In cell D28 one writes the equation: `=SUM(D4..D26)`, which will calculate the sum of the thermal resistances from the rows above. Similarly should be done for the sum of diffusion resistances in column F.

A steady state calculation method will be used to determine the temperature distribution through the construction - this is the same as to say that the conditions in the indoor and outdoor surroundings have been the same for a long time. By the calculation one should distribute the total temperature drop over the whole construction over the individual layers according to their share of the total thermal resistance. Starting from the outside, the temperature of the exterior surface can be calculated as:

$$\theta_{ext.\,surf.} = \theta_{out} + \frac{R_{ext.\,surf.}}{\sum R_{all\ layers}} (\theta_{in} - \theta_{out}) \qquad (1)$$

The general equation for an interface will be:

$$\theta_{i\,/\,i+1} = \theta_{i-1\,/\,i} + \frac{R_i}{\sum R_{all\ layers}} (\theta_{in} - \theta_{out}) \qquad (2)$$

So each temperature is calculated on the basis of the previous, and this is continued all the way to the inside surface of the construction. The temperatures will be calculated in column G, and the temperature for the outside of the first layer will then be in cell G5. Assuming that row 3 represents the outdoor conditions, and row 27 the indoor conditions, the temperature can be calculated as:

```
=G3+D4/D$28*(G$27-G$3)
```

Note the $-signs in front some of the row number references. These signs mean nothing for the calculation of the actual cell, but when the equation is copied to the other cells in the same column, the principle about same relative position of the cells called will not hold for the cells whose row number is referred with a $-sign. Such row numbers are fixed in the spreadsheet's system for absolute indexing, and will not change by copying. When the formula is copied to e.g. cell G7, the result becomes:

```
=G5+D6/D$28*(G$27-G$3)
```

A similar principle can be used by putting a $-character in front of the column index, which by copying between columns would mean that the column reference is static.

Equations like two shown above should be copied to all odd-numbered rows in the temperature column, except the rows for the outside and inside boundary conditions. It is now possible to make a drawing of the temperature distribution using the spreadsheet program's graphical functions. For this, one will have to make supplementary column which will contain the x-coordinates of the interfaces of the layers. Place the column somewhere to the right in the spreadsheet. This column is made by giving the outer boundary the coordinate 0, and finding the other co-ordinates by successively adding the thicknesses of the layers ($x_{i/i+1}$ is equal to $x_{i/i-1} + d_i$). For this purpose, one could (fictitiously) give the inside and outside boundaries a thickness of for instance 0.02 m. The X-and Y-series should start in the same row, an dthen the temperature profile can be seen as an XY-graph. This is done in Figure 2. With this chart it is possoble only to get the temperatures represented with symbols without connecting lines. This problems will be dealt with towards the end of this note.

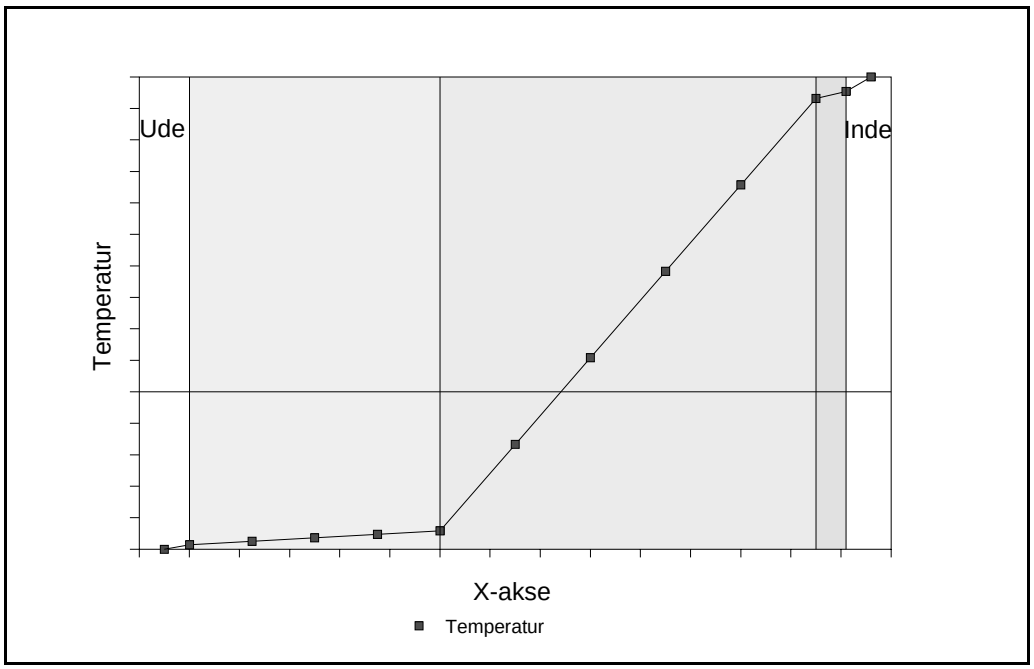


Figure 2 Drawing of the temperature distribution through a construction consisting of three materials with different thermal conductivity.

We will now determine the moisture profiles. It is required first to know the saturation value of the vapour pressure in the interface of each layer. Traditionally, this is done by looking for the values in a table, but this is not feasible here where we want an automatic update of the spreadsheet if for instance a thermal boundary condition is changed. However, there exists some approximate formulas that give the saturation vapour pressure as a function of the temperature. One such formula is:

$$p_s = \exp\left(23.5771 - \frac{4042.90}{\theta + 235.57}\right) \qquad [Pa] \qquad \textbf{(3)}$$

In this equation, the temperatyure should be entered in °C. A p_s column can now be created to the right of the temperature column. The exponential function is called in the spreadsheet either as `=EXP()` or `=EKSP()`.

A column should be created also for the relative humidity, but leave this column empty, except for the cells that represent the air on each side of the construction - the boundary conditions of the problem.

It is now possible to make a first calculation of the vapour concentration profile through the construction. First, the inside and outside water vapour pressures are determined each of the two places as the relative humidity (as a pure number) times the saturation vapour pressure. Afterwards, the vapour pressure is determined at all the internal material interfaces. This is done according to similar equations as for the calculation of the temperature distribution, while the inside and outside *p*-values replace the inside and outside temperatures, and *Z*-values replaces thermal resistances. *Z*-values are taken from column F, and the *p*-profile is calculated in columns J (as illustrated here for cell J5):

```
=J3+F4/F$28*(J$27-J$3)
```

This equation is copied to all the other interfaces in the J-column. Now also the profiles for saturation vapour pressure and the first determination of the vapour pressure can be drawn graphically. This can be done in two ways:

(1) Either in a traditional diagram with the *x*-coordinate along the abscissa-axis. This is done simply by choosing p_s and p as new series in the temperature diagram from before (og omdøbe diagrammet); se fodnote 1 side 5..
(2) Alternatively, the water vapour diffusion resistance cumulated from layer to layer could be used as abscissa. In such a diagram, diffusion open layers (such as mineral wool) will appear as "thin" layers, while vapour tight layers (such as vapour retarders) will appear "thick". This type of diagram could be preferred for our use, as the slope of the vapour pressure profile indicates the rate of water vapour diffusion, $\Delta p/\Delta Z$. On will have to create a column with the Z-values, where the values are found by successively adding the diffusion resistance of the layers. This column will be used as the abscissa of the graph.

If all the vapour pressures are below the saturation vapour pressure, then there is no condensation in the construction, and the p-profile found is then the final distribution. To evaluate this, one should also consider whether there could be areas between the material interfaces, where the p- and p_s-curves intersect - remember the p_s-curve is not straight. If this is suspected, the calculation could be improved by dividing the calculation into some thinner layers. For these calculations, it could therefore be advantageous to divide thick layers into several thinner layers.

If the vapour pressure profile crosses the saturation profile, it means that condensation appears at least one place in the construction within the region where the profiles overlap - but not necessarily in the whole region. An example of this is shown in Figure 3. Vapour pressures higher than the saturation values are not physically possible, so the vapour pressure profile just found, cannot be correct and will have to be amended, sp the curve just follows the saturation curve tangentially.

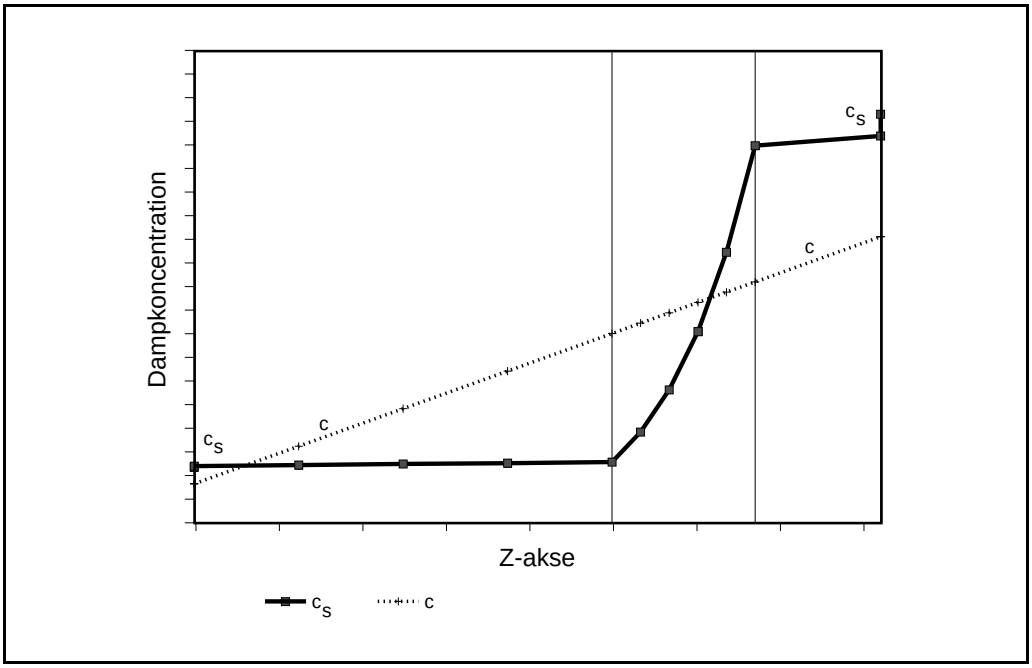


Figure 3 The distribution of vapour pressures vs. *Z*-value according to the first calculation.

Since our aim is to set up a spreadsheet that can be used generally, we must consider the situation that condensation may occur. This will be done in the section.

The corrected vapour concentration profile

When the vapour pressure is drawn graphically with the *Z*-value as abscissa, it is easy to see if the vapour pressure profiles cross each other, and if that is the case to construct a corrected profile starting and ending at boundary conditions' vapour pressures, and which inside the construction just touches the saturation profile tangentially. If condensation occurs, the vapour pressure profile will have a larger slope where teh vapour comes from (the upstream side) than on the downstream side. Since the slope of the profile indicates the rate of vapour flow, the amount of condensation can be read as the change of slope of the vapour pressure profile over the region where condensation takes place.

In a spreadsheet however, we have to construct the corrected vapour pressure profile using equations. This is done using two intermediate vapour pressure variables, *p1* and *p2*. When we constructed our first prediction of the vapour pressure profile, we used the vapour pressure outdoors, and for each layer we calculated an increment in the vapour pressure over that layer by distributing the total vapour pressure difference over the construction according to each layer's relative share of the total vapour diffusion resistance (ΣZ) of the whole construction. Thus, we assumed the vapour flow, *g*, which could be calculated as $(p_{in} - p_{out})/\Sigma Z$, would be the same through the whole construction, so Δp_i could be calculated as $Z_i \cdot g$. However, *g* is not the same through the whole construction if condensation occurs.

Two new columns, e.g. L and M are used in the spreadsheet for the intermediate vapour pressures *p1* and *p2*. The *p1*-profile is calculated partly in the same way as the uncorrected *p*-profile, but with a continuous check that the calculated *p* does not exceed p_s. When, in this way, one has found the value $p_{i/i+1}$ for the interface between layers *i* and *i+1* insider the construction, this value is now used as the left-hand boundary for calculating $p_{i+1/i+2}$, while $p_{indoors}$ is used as the right-hand boundary. $p_{i+2/i+3}$ is now calculated with $p_{i+1/i+2}$ as left-hand boundary condition - and so forward, always with $p_{indoors}$ as the right-hand boundary condition. In this way will be achieved results, which always attempt to construct a straight line from the vapour pressure calculated in the previous interface to the vapour pressure indoors, but not exceeding the saturation vapour pressure. The result is illustrated in Figure 4. The spreadsheet formula to calculate *p1* in for instance row 5 (the exterior boundary condition) becomes, since the *p1*-values re calculated in column L, p_s-values are found in column H, *Z*-values in column F, and the indoor conditions in row 27:

```
=MIN( L3 + F4/SUM(F4..F$26)*(L$27-L3); H5 )
```

The blank spaces in this algorithm are only shown here to improve the legibility. They must not be there in the spreadsheet. The conscious way $-signs are used in the algorithm means that the outward boundary condition always "floats" dynamically as the previous interface condition before the interface that is under current calculation. The equation can immediately be copied to the cells L7, L9, …, L25. It is prsumed that the cells L3 and L27 contain the outdoor and indoor boundary conditions. This is ensured for instance in L3 by writing: =J3.

The resulting vapour pressure curve has an area downstream from the condensation area where it is concave downward (it's second derivative is negative). This means that more moisture flows away from this area, than what is flowing to the area, and this cannot be possible in a steady state situation. But upstream from the condensation area we have now obtained a determination of the curve which, starting from the indoor conditions just becomes a tangent to the p_s-curve. So now we have to find the vapour pressure distribution in the downstream area.

p2 is determined in the same way as *p1*, but this time starting from the inside conditions, and all the time attempting to construct a linear vapour pressure distribution towards the outside conditions, while the inward boundary conditions keeps "floating" after the calculated *p*'s in the outward direction. The *p2*-values are calculated in column M. The eqauatoin to do this for cell M25 (den innermost interface) becomes:

```
=MIN( M27 - F26/SUM(F$4..F26)*(M27-M$3); H25 )
```

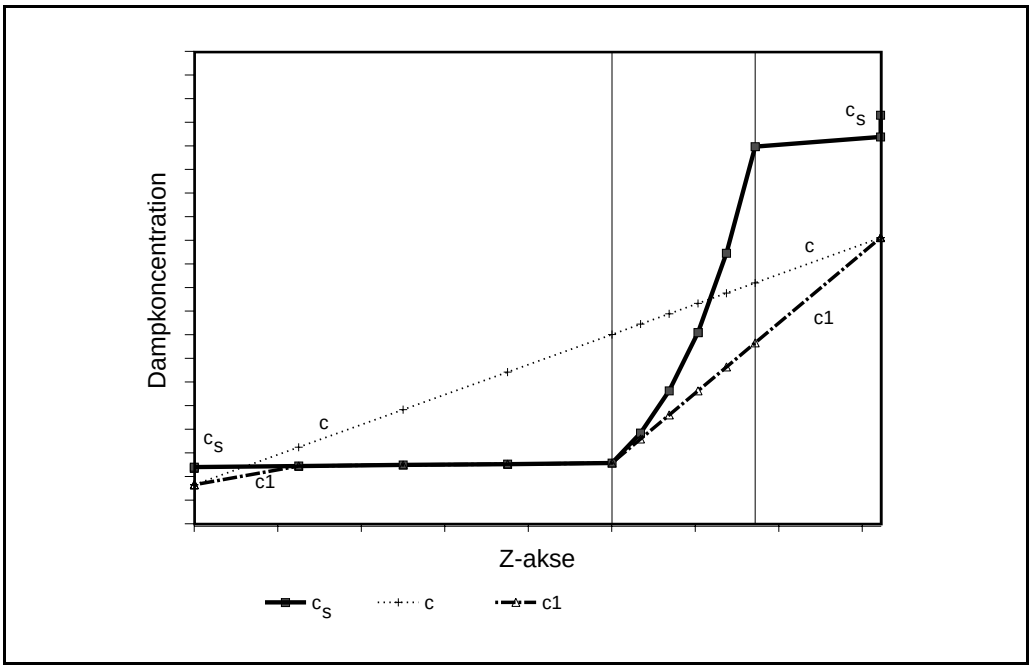


Figure 4 Drawing of the vapour pressure profile *p1*.

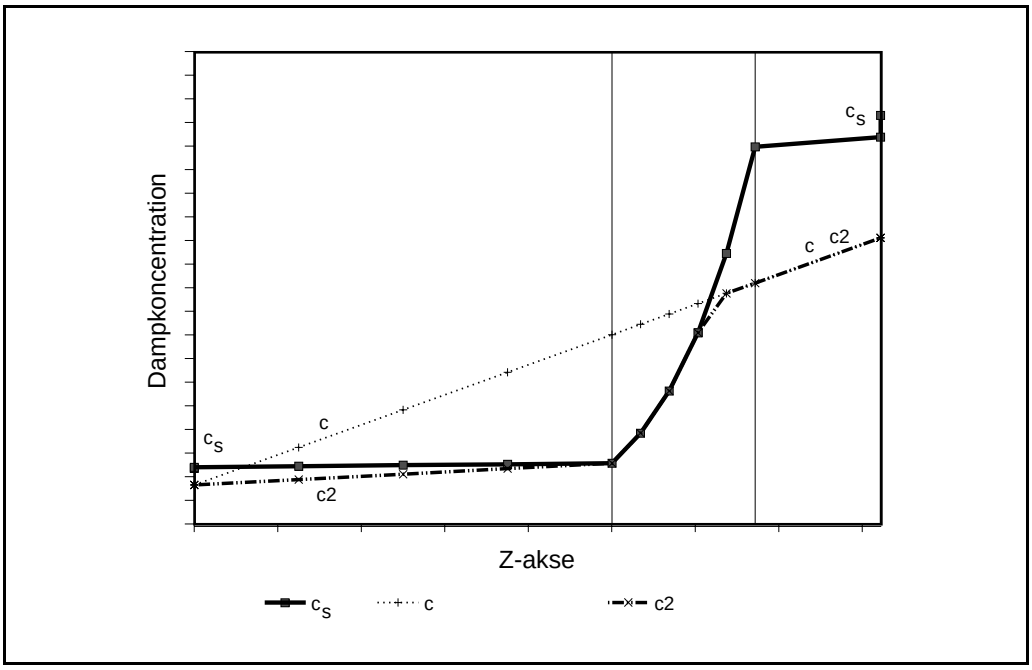


Figure 5 Drawing of the vapour pressure profile *p2*.

The equation can be copied to M23, M21, …, M5. In M3 and M27 should be the values from J3 and J 27, respectively. The resulting p2-profile is shown in Figure 5. From this profile, the results from the downstream side of the condensation area can be used to describe the corrected vapour pressure profile.

In every interface, the point for the corrected vapour pressure profile is now found as the minimum value of *p1* and *p2*. This is calculated in the until now not used column K, so for instance K5 becomes:

```
=MIN(L5;M5)
```

The corrected vapour pressure profile is drawn in Figure 6, and one sees how it runs as the shortest possible connecting line between $p_{outdoors}$ and $p_{indoors}$, that does not intersect, but is only a tangent to the saturation pressure curve.

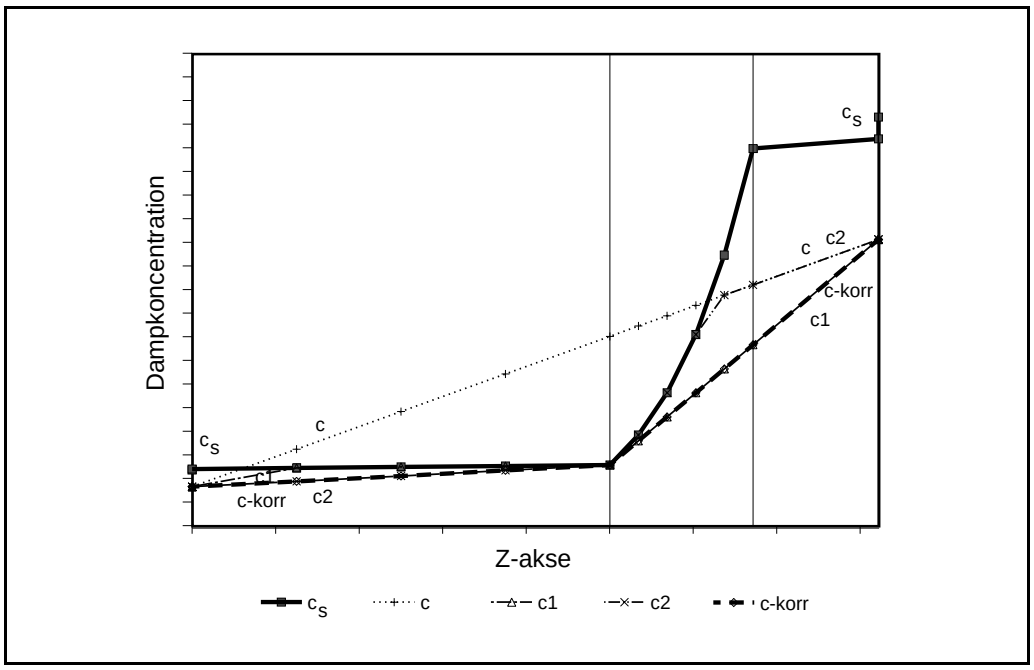


Figure 6 Drawing of all intermediate vapour pressure profiles along with the corrected profile.

RH-values for the interfaces between the layers can now be found in the still not used inner cells of column I. RH is determined as the corrected vapour pressure divided by the saturation vapour pressure. E.g. for cell I5: `=K5/H5`.

The RH-values should show be less than or equal to 1.0. Now also the RH-profile can be drawn.

Finally, one could introduce a new column, e.g. column N, where the vapour flow through each layer in the construction could be calculated. This is calculated as the difference in corrected vapour pressure over each single layer, divided by its vapour diffusion resistance (*Z*-value). By comparing the vapour flow running into the construction with the vapour flow that runs out, one will have a calculation of the total amount of interstitial condensation in the construction. This result is equivalent to the one that could be read graphically from the slopes of the p_{corr} curve in the diagram that has the *Z*-values as abscissa.

Calculation of drying

The description above is valid for calculation of the condensation risk of a construction which for "infinite" time has been subjected to the same climatic conditions. In the calculations was assumed that one had no a priori knowledge about where in the construction condensation would occur. Sometimes however, it is necessary to make calculations of the situation when one knows that there is already condensed water in a construction, and one wants to investigate the possibilities for this water to dry out when the climatic conditions are amenable. This way, it will be possible to predict a potential drying rate.

This is done according to the same principle as above, but in the layer where free water is present one will the values of p, $p1$, and $p2$ to be equal to p_s. One should now regard the calculations to each of the layer with free water as two separate calculations according to the same procedure as above. In each half one should insert as one of the boundary conditions, the one for the inner layer where the known condition: Free water (100 % RH) exists.

As an example will be shown the calculation of the uncorrected vapour pressure in cell J5 when it is assumed that the interface in row 13 contains free water that should dry out. Like before, rows 3 and 27 contain the data for the outdoor and indoor conditions, respectively.

```
=J3+F4/SUM(F$4..F$12)*(J$13-J$3)
```

This algorithm can be copied to J7, J9 and J11. On the inside of the wet layer, the vapour pressure is calculated for instance in row 15 as:

```
=J13+F14/SUM(F$14..F$26)*(J$27-J$13)
```

This algorithm can be copied to J17, J19, …, J25. An example of the vapour pressure profiles during drying can be seen in Figure 7. In the example is assumed tha the temperature is the same on both sides of the construction.

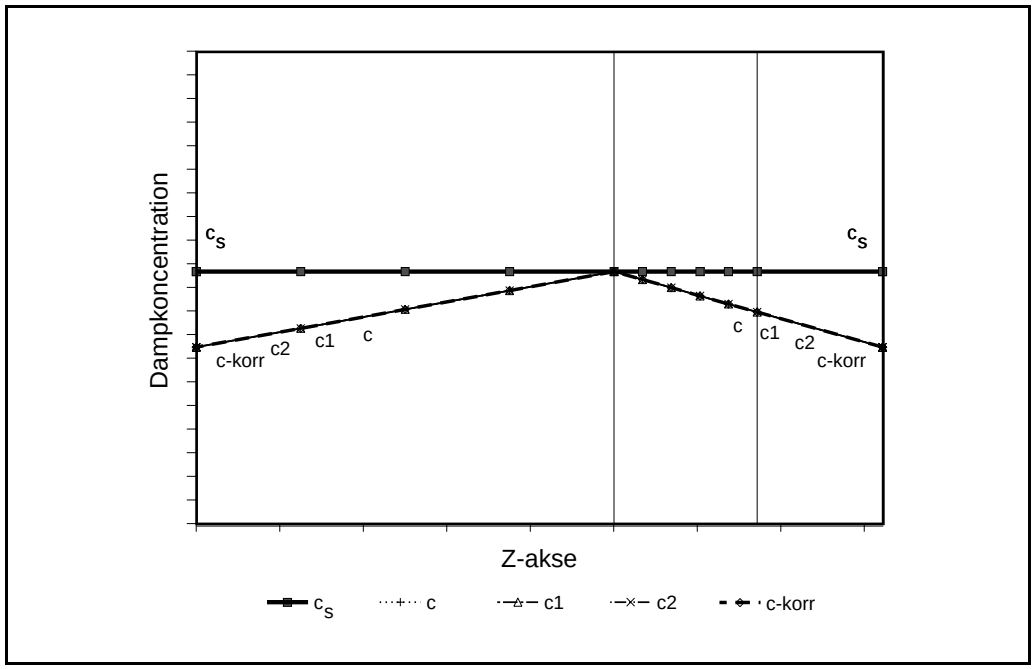


Figure 7 Drawing of the vapour pressure profiles with drying from a layer that is wet from the beginning. All corrected and uncorrected p-profiles are identical in this case.

APPENDIX

Drawing of graphs with connected lines between the points

In the graphs of this lecture note are used points to represent the results in interfaces between layers, and lines to connect the points. With the graphics of spreapsheet programs it is only possible to get the lines drawn if there are no empty rows between the rows that hold the data to be plotted. Unfortunately the set-up advised in this lecture note has exactly this feature: The data come in each second row, and are therefore intersected by empty rows. The intent was that it should be easy to see which data concern the material layers, and which concern the interface between layers.

If one could imagine this differently, one could devise another set-up where only about half as many rows are needed without empty rows in between. All values concerning interfaces just have to be shifted on row up, and then one can delete the empty rows that now become redundant.

Another possibility is to make a new section somewhere else in the spreadsheet where the values that are to be shown graphically are replicated in a continuous form, row after row. The =-operator can be used in a cell to always keep it instantly updated with values from another cell. Use such a section to work as an echo of the spreadsheet that was described in the previous sections, but formatted in such a way that the graphs can be drawn without problems.